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FREQUENCY OF OCCURRENCE OF CRITICAL GUST LOADS ON OVERLOADED AIRPLANES

By Thomas D. Reisert

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

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ADVANCE RESTRICTED REPORT

FREQUENCY OF OCCURRENCE OF CRITICAL GUST LOADS ON OVERLOADED AIRPLANES

By Thomas D. Reisert

SUMMARY

Statistical gust-frequency data taken by the National Advisory Committee for Aeronautics were used to determine the effect of overloading an airplane on the frequency of occurrence of gust loads that stress the airplane to or above its design applied bending moment. The analysis was made for three transport-type airplanes operating at cruising power with overloads varying from 0 to 50 percent of the design gross weight. The results are presented in the form of curves of critical gust-load frequency as a function of overload with design gust velocity and wing-weight ratio as parameters.

The probability of structural failure was found to increase rapidly with overload. The effect on the critical gust-load frequency of the distribution of overload weight was of equal or greater importance than the effect of the amount of overload. For different airplanes the change in critical gust-load frequency for the same overload condition was concluded to be due to the variation in the ratio of design wing weight to design gross weight and to the variation in mean wing chord.

INTRODUCTION

Overloading airplanes to increase the pay load or range has been a common practice during the war. This overloading increases the stresses in the structure at unit load factor and requires more care in operations than normal loading.

The total applied load for which an airplane structure is designed is composed of a steady load, or lift, equal to the weight of the airplane and a load increment applied by an atmospheric gust or by movement of the control surfaces by the pilot. When the gross weight is increased, the allowable increment of load is obviously decreased. A gust of smaller intensity than that designed for will therefore result in limit load and, since the smaller gusts tend to occur more frequently than the larger gusts, the chances of failure increase.

In order to determine the effect of overload on the frequency of occurrence of stresses equal to or in excess of the design yield stress, the results on the frequency of occurrence of gusts presented in reference 1 were applied to analyses of three transport-type airplanes with overloads varying from 0 to 50 percent of the design gross weight. The present report gives the results of these analyses and shows the significance of airplane characteristics and manner of overloading on the frequency of occurrence of critical stress.

SYMBOLS

n	load factor
n_a	allowable applied load factor
n_d	design applied load factor
Δn	increment of load factor
W	operating weight of airplane, pounds
W_d	design gross weight, pounds
W_w	design wing weight, pounds
K	relative alleviation factor
U_e	effective gust velocity, feet per second
m	slope of lift curve, per radian
S	wing area, square feet
W/S	wing loading, pounds per square foot

$\bar{c}$ mean wing chord, feet

f frequency, number of occurrences of a phenomenon within a class interval

Σf_r ratio of number of occurrences within and exceeding a class interval to the total number of occurrences in a sample

f_r relative frequency of gusts of different intensity, ratio of number of occurrences of a phenomenon within a class interval to total number of occurrences (reference 1)

$V\sigma^{1/2}$ equivalent airspeed, miles per hour

V_L maximum equivalent airspeed in level flight, miles per hour

METHOD

General Procedure

The method consisted in determining the frequency of the gust that, for the overloaded airplane, would apply a net wing bending moment equal to or greater than that corresponding to the design applied load. This frequency, which is hereinafter designated critical gust-load frequency, was found by three steps: (1) determination of the operating speed of the overloaded airplane at an assumed cruising power; (2) determination of the equivalent design gust velocity, that is, the gust velocity that would subject the wing to its design bending moment when the overloaded airplane is operating at this speed; and (3) determination of the frequency with which the overloaded airplane would encounter gusts equal to or greater than gusts of this intensity.

In the determination of the operating speed of the overloaded airplane, the airplane has been assumed to fly during its entire life at a speed obtained with cruising power, which was taken as the power required to maintain $0.8V_L$ at normal gross weight. The computation of the effect of overload on airplane performance was made on the assumption that the only performance characteristic which varies with loading is the power required to overcome the change in induced drag. The results of a sample computation (fig. 1) were determined by selecting the airspeeds corresponding to cruising power on power-required curves for several conditions of overload. The assumptions used appear to be sufficiently valid for the purpose of the present analysis. For any specific case, however, detailed calculation might be made to include propeller and engine characteristics and the effect of changes in the parasite drag on the required power. Operations under special conditions, such as at low speed to obtain optimum range, would obviously be an important consideration. The computations, however made, should give a plot of the operating speed as a function of overload.

The magnitude of the gust velocity that subjects the wing to its design bending moment depends on the distribution of the overload weight in the airplane. The analysis was therefore made for two limiting conditions of overload distribution. For the first condition (case I) the weight distribution of the overloaded airplane was assumed to be the same as that for the condition of normal gross weight. For the second condition (case II) all overload weight was assumed to be concentrated in the fuselage. The actual overload distribution of any airplane would probably fall between these limits and would more than likely be closer to case II.

In case I, in which the stress for unit load factor is proportional to the gross weight of the airplane, the allowable applied load factor is given by the expression

$$n_a = \frac{W_d n_d}{W} \tag{1}$$

The use of equation (1) implies that equal loads cause equal bending moments and that the relation is independent of the angle of attack of the airplane. The value for n_d is determined on the basis of the design specifications in reference 2.

For case II the following formula based on the root bending moment of the wing has been derived to obtain the allowable applied load factor:

$$n_a = \frac{n_d(W_d - W_w)}{W - W_w} \tag{2}$$

Inasmuch as the wing weight does not change in this case, the relative alleviating effect of wing inertia is reduced in proportion to the ratio of actual weight to design gross weight. In the derivation of equation (2) the centroid of the air-load distribution on the wing and the centroid of the wing weight were assumed to coincide.

In order to obtain the equivalent design gust velocity, the increment of the allowable applied load factor $n_a - 1$ and the operating speed previously computed for the overloaded airplane were substituted in the formula

$$\begin{aligned} n_a - 1 &= \Delta n \\ &= \frac{KU_e V\sigma^{1/2} m}{575\frac{W}{S}} \end{aligned} \tag{3}$$

In equation (3) the factor K should be the value, obtained from figure 11(a) of reference 2, that corresponds to the wing loading for the overloaded airplane.

The results of reference 1 were used to determine the number of times a gust of intensity equal to or greater than the value computed would be encountered. The use of these data involves the selection of a relative distribution of gust intensities and an estimate of the total number of gusts of all sizes that may be encountered. Figure 7 of reference 1 presents two summation curves of gust-frequency distribution, curves A and B, which correspond to the upper and the lower limits of the data, respectively. Summation curve A, which is reproduced herein as figure 2, has been selected as the basis for the present analysis. This curve shows the fraction of the total frequency with which a gust of given intensity will be equaled or exceeded.

It should be noted that the values of Σf_r taken from the curve of figure 2 were multiplied by 2 in order to take into account both positive and negative gusts. In the actual case, however, the factor will vary from 2 to 1 as the overload increases. If the airplane were designed for limit loads corresponding to those imposed by gust velocities of ±30 feet per second for normal gross weight, then, to a first approximation, the difference between positive and negative limit loads would correspond to the sum of the design gust velocities, about 60 feet per second. As the positive critical gust velocity decreased from 30 to 20 feet per second, the negative critical gust velocity would change from -30 to -40 feet per second and the total frequency of critical gusts would equal the sum of the gust frequencies for 20 and 40 feet per second instead of twice the gust frequency for 20 feet per second. The limit would be reached when the negative critical gust velocity is so large that the corresponding gust frequency would be 0 and the factor under consideration would be 1. The comparison indicates that the two methods will yield equal gust frequencies at zero overload but, as the overload increases, the assumption of a constant factor of 2 will be conservative.

For an airplane with a mean chord of 10 feet, the value of total gust frequency used in the present analysis was taken as 5 gusts for a flight path of 1 mile. This value corresponds very nearly to the total frequency reported in reference 1, which represents average conditions for several types of transport operation. Multiplying by the ratio of mean chords $10/\bar{c}$ results in a value of $50/\bar{c}$ gusts per mile for any airplane.

In order to determine the number of times the equivalent design gust velocity is equaled or exceeded, this velocity is used in figure 2 to obtain Σf_r. The critical gust-load frequency for a flight path of 1 mile is then

$$f = 2\,\Sigma f_r \frac{50}{\bar{c}}$$

In the present report the value so determined will be multiplied by 10^6 to obtain the more convenient unit of gusts per million miles of operation.

Conditions Selected for Analysis

The analysis was made for three transport-type airplanes of different size: the Douglas DC-3, the Boeing S-307, and a hypothetical airplane of large size. The characteristics of these airplanes are given in the following table:

	Douglas DC-3	Boeing S-307	Hypothetical airplane
Gross weight, lb	24,000	45,000	65,000
Wing loading, lb/sq ft	24.3	30.2	38
V_L, mph	213	250	300
Slope of lift curve, per radian	4.79	4.66	5.0
Wing area, sq ft	988	1490	1710
M.A.C., ft	10.4	13.85	12.22
Wing span, ft	95	108	140
Design applied load, lb	80,900	147,000	225,000
Estimated wing weight, lb	13,800	27,350	45,500
Wing-weight ratio	0.575	0.608	0.700
Design applied load factor	3.37	3.27	3.46

The design strengths of these airplanes were assumed to correspond to the usual gust design requirement for the condition of normal gross weight (reference 2) with the design speed taken as the maximum equivalent airspeed in level flight V_L. The analysis was, however, carried out for strengths equal to 90, 100, and 110 percent of the design value to include the effects of understrength, required strength, and overstrength.

Sample computations of critical gust-load frequency for the Boeing S-307 airplane overloaded 20 percent for both cases I and II are given in the appendix. A series of calculations were also made for case II in which the wing-weight ratio was assumed to vary from 0 to 0.8.

Although the usual gust design requirement is based on an effective gust velocity of 30 feet per second, smaller values are sometimes used. In order to show the effect of changes in design gust velocity on the variation of critical gust-load frequency with overload, the analysis was made for the Boeing S-307 airplane with the overload distribution specified for case I, in which the design gust velocity was varied from 20 to 35 feet per second. The result of this calculation is essentially the same as the calculations in which the design strength was assumed to vary by fixed percentages; for this calculation the range of difference in design strength is greater and the results are expressed in terms of change in design gust velocity.

RESULTS AND DISCUSSION

The results of the analysis are shown in figures 3 to 5. Figure 3 indicates that the critical gust-load frequency and, consequently, the probability of structural failure in rough air increases rapidly with overload. For case I (fig. 3(a)), a 10-percent overload increases the critical gust-load frequency by a factor of 1.7 and a 20-percent overload increases the frequency by a factor of 2.6. It will be noted in figure 3(a) that the corresponding curves for the three airplanes are approximately parallel but are somewhat displaced. This displacement is due to the variation in total gust frequency with airplane size or mean wing chord.

Figure 3(b) shows that the critical gust-load frequency varies considerably for different airplanes and increases very rapidly with overload when the overload is concentrated in the fuselage (case II). The curves for the Boeing S-307 airplane, for example, show that a 10-percent overload would increase the critical gust-load frequency 10 times that for normal gross weight and that a 20-percent overload would increase the critical gust-load frequency 78 times that for normal gross weight. Comparison of figures 3(a) and 3(b) indicates that the load distribution has a pronounced effect which is at least as important as the amount of overload. The displacement of the curves for the three airplanes that results from difference in size, as noted in figure 3(a), is also present in figure 3(b) but is noticeable only at zero overload, the effect being obscured by other factors with overload.

Since the critical gust-load frequency is the frequency of gusts having an intensity equal to or exceeding that required to apply yield-point load, it is evident that any frequency greater than unity may cause failure. The actual frequency that will cause failure is not known; however, if some frequency is assumed, the operating mileage per failure can be estimated for any amount of overload. Conversely, the amount of overload that may be expected to result in failure in a given number of operating miles may be determined. For example, from the curve in figure 3(a) for the Boeing S-307 airplane built to withstand 100 percent of the design applied bending moment, an assumed critical frequency of 3 would result in failure in 1,000,000 miles of operation with 38-percent overload for case I. For case II (fig. 3(b)), the corresponding value of overload is only 7 percent for one failure in 1,000,000 miles of flight.

Figure 4 indicates that the critical gust-load frequency varies radically for different wing-weight ratios. The critical gust-load frequency as predicted by this analysis becomes extremely large for case II at high wing-weight ratios and is absurdly high for the extreme case of a wing-weight ratio of 1 (the flying wing). For example, an airplane with a wing-weight ratio of 0.80 overloaded 12 percent may be expected in 1,000,000 miles of flight to encounter 180 critical gusts, each of which will stress the airplane to or above its design applied bending moment. This number of critical gust loads is extremely high and, consequently, the airplane life would be shortened considerably if flown in this condition. For extreme cases such as these, the critical design condition would probably be something other than the gust condition - that is, landing loads - although, even in the case of the hypothetical airplane used herein having a wing-weight ratio as high as 0.70, the gust condition is more critical than the landing condition as defined by present design requirements. It is evident that caution must be used when these methods are applied to extreme types of aircraft.

The curves of critical gust-load frequency given in figure 5 show how the probability of structural failure due to gust loads changes as the design gust velocity is changed. The relative displacement of the curves of figure 5 for different design gust velocities is more significant than the absolute value for a given condition. As shown in figure 5, an increase of 5 feet per second in the design gust velocity results in a decrease in the critical gust-load frequency to one-sixth of the original value. For the conditions assumed for the Boeing S-307 airplane, such an increase in design gust velocity also results in a 40-percent increase in allowable overload for a given gust-load frequency.

In the present analysis, the airplane has been assumed to operate at a given overload throughout its life. This assumption is obviously not true, because the amount of overload will vary continuously during a given flight as well as for different flights. It is also probable that, in general, the time that the airplane is overloaded would be relatively small. The absolute values of critical gust-load frequency calculated herein may thus be considered somewhat conservative. It should also be pointed out that, because of the assumption of an operating speed based on cruising power, the absolute values of critical gust-load frequency are high as compared with values that would result from an assumption of reduced speed in rough air. If the procedure of the present report is used with a speed 75 percent of the cruising speed, the frequencies are reduced to approximately one-hundredth of the frequencies at the cruising speed.

CONCLUDING REMARKS

An analysis of the effect of overloading an airplane on the frequency of occurrence of gusts that stress the airplane to or above its design applied bending moment has been made for three transport-type airplanes operating at cruising power and with overload varying from 0 to 50 percent of the design gross weight.

The probability of structural failure of an airplane resulting from gust loads was considerably increased with moderate overloads. The distribution of the overload weight in the airplane had an important effect on the probability of failure. The concentration of overload in the fuselage showed a pronounced increase in probability of failure over the proportional distribution of overload throughout the airplane. For example, a cargo or fuselage overload of 10 percent in the Boeing S-307 airplane increased the probability of failure about 10 times as compared with the normal load condition, whereas the same overload proportionally distributed increased the probability of failure 1.7 times. For a 20-percent overload the probability of failure is increased

78 times for a fuselage overload and 2.6 times for the proportional distribution.

The effect of wing-weight ratio (the ratio of the design wing weight to the design gross weight) on the probability of failure with overload in the fuselage was also important, particularly for airplanes with high wing-weight ratios. The probability of failure also varied to some extent with mean wing chord of the airplane.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

APPENDIX

SAMPLE COMPUTATIONS OF CRITICAL GUST-LOAD FREQUENCY

The critical gust-load frequency is calculated herein for the Boeing S-307 airplane operating under the following conditions:

Overload, percent gross weight	20
Operating weight of overloaded airplane, lb	54,000
Operating speed at cruising power for 20-percent overload, mph	195

For case I (overload weight distributed proportionally throughout airplane), the equivalent design gust velocity is found as follows:

From equation (1),

$$n_a = \frac{45,000 \times 3.27}{54,000}$$

$$= 2.72$$

From equation (3),

$$\Delta n = n_a - 1$$

$$= 1.72$$

and the equivalent design gust velocity

$$U_e = \frac{575 \times 1.72 \times 36.2}{1.15 \times 195 \times 4.66}$$

$$= 34.3$$

The frequency with which the overloaded airplane would encounter gusts equal to or greater than gusts of this intensity is found from figure 2 to be

$$\Sigma f_r = 2.05 \times 10^{-7}$$

from which,

$$f = 2 \times \frac{2.05}{10^7} \times \frac{50}{13.85} \times 10^6$$

$$= 1.48 \text{ gusts per million miles}$$

Similarly, for case II (overload weight concentrated in fuselage),

$$n_a = \frac{3.26(45,000 - 27,350)}{54,000 - 27,350}$$

$$= 2.165$$

$$\Delta n = 1.165$$

$$U_e = \frac{575 \times 1.165 \times 36.2}{1.15 \times 195 \times 4.66}$$

$$= 23.2$$

$$\Sigma f_r = 5.4 \times 10^{-6}$$

$$f = 2 \times \frac{5.4}{10^6} \times \frac{50}{13.85} \times 10^6$$

$$= 39.0 \text{ gusts per million miles}$$

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2. Anon.: Airplane Airworthiness. Pt. 04 of Civil Aero. Manual, CAA, U. S. Dept. Commerce, Feb. 1, 1941, pp. .2-1 - .2-7.

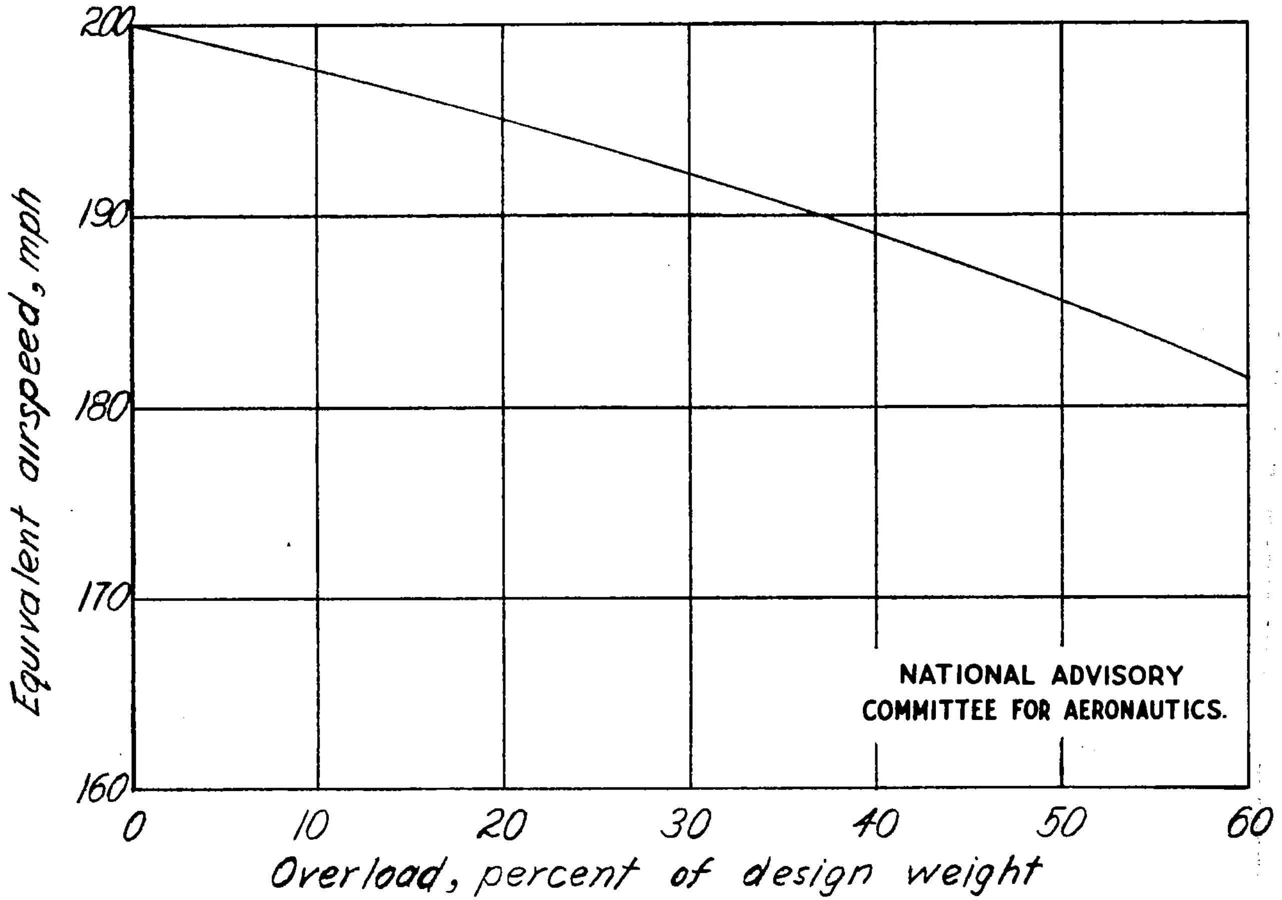


Figure 1.– Variation of airspeed with overload.
Boeing S-307 airplane; cruising power.

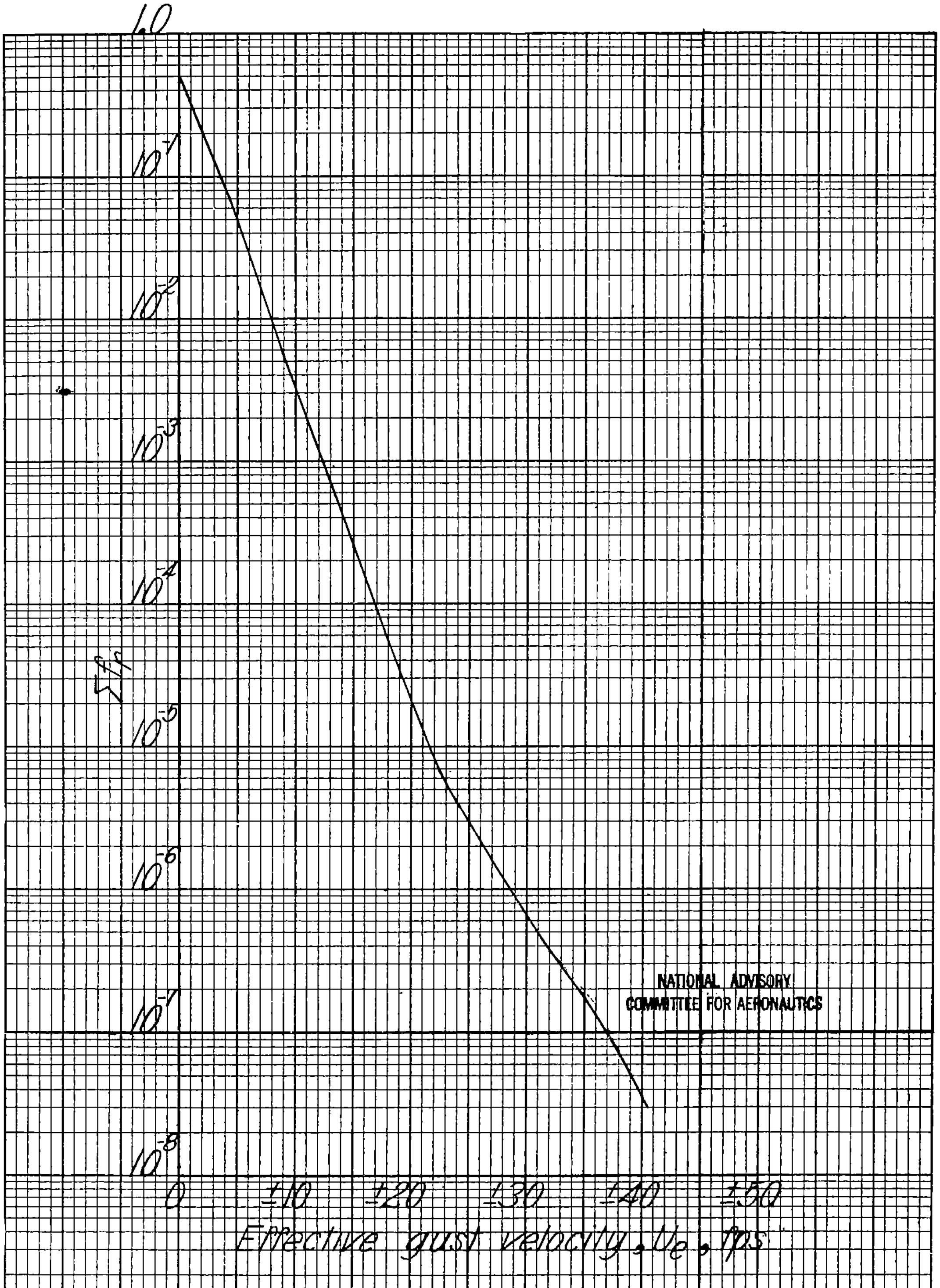
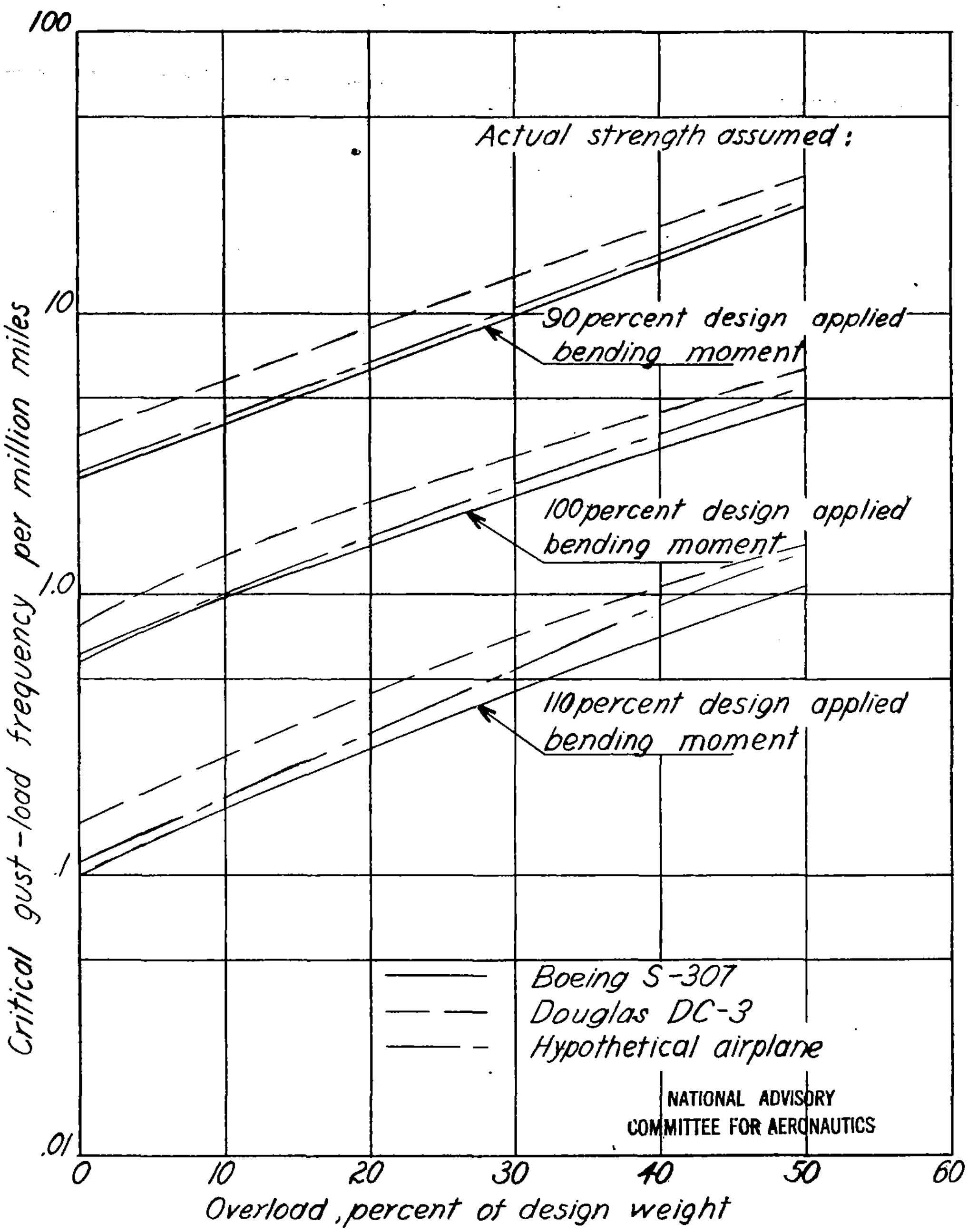
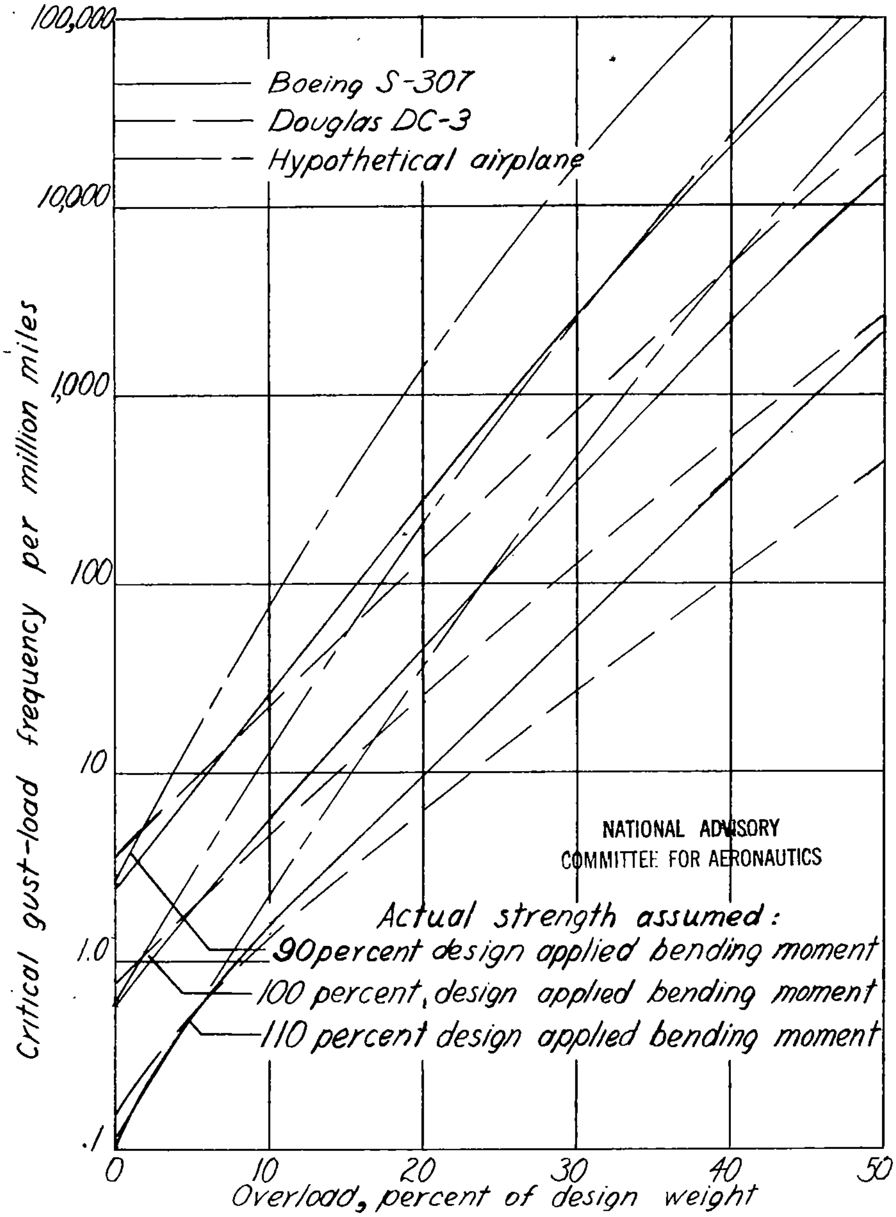


Figure 2.- Unit summation curve defining approximate maximum limit of samples. (From figure 7 of reference 1.)



(a) Case I.

Figure 3 - Variation of critical gust-load frequency with overload for three strength conditions.



(b) Case II.

Figure 3.- Concluded.

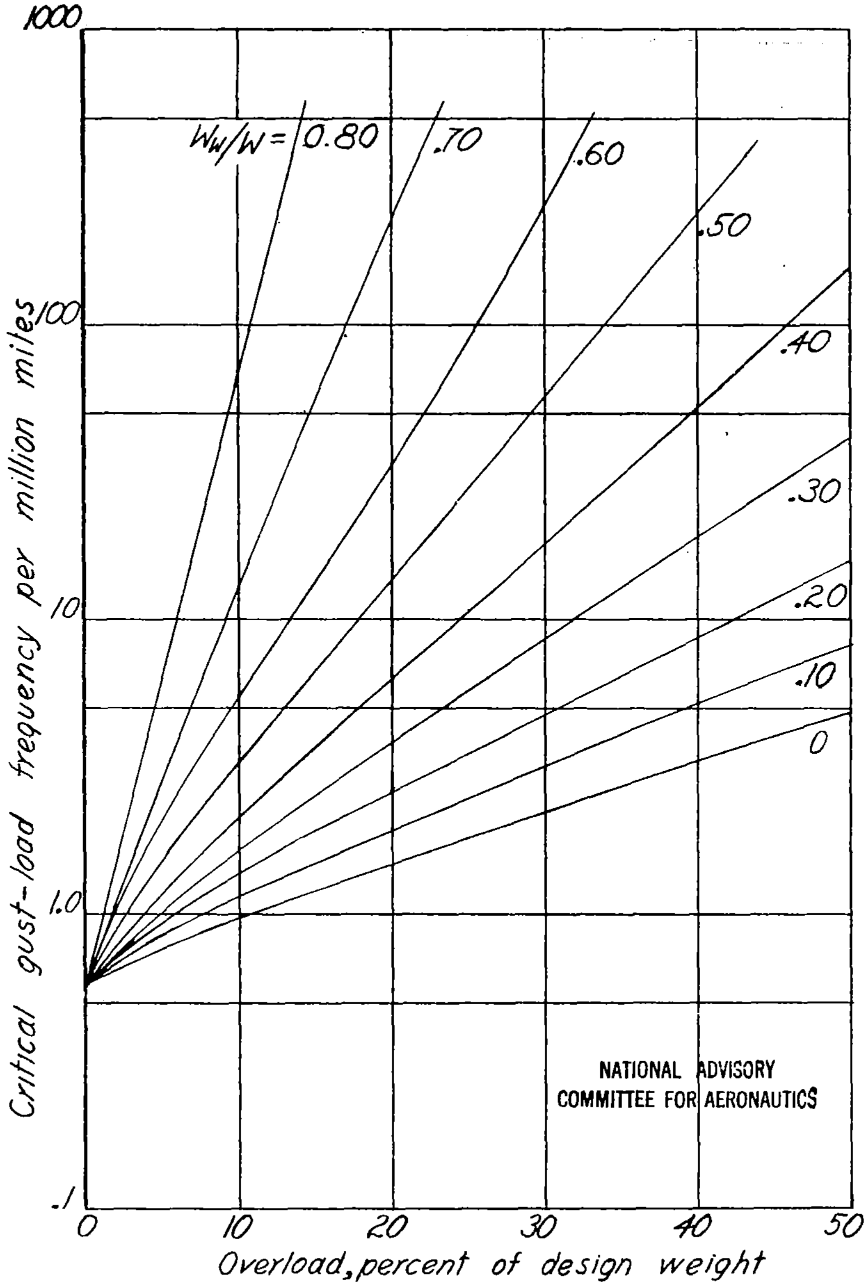


Figure 4.- Effect of wing-weight ratio on the critical gust-load frequency as a function of overload. Boeing S-307 airplane; case II.

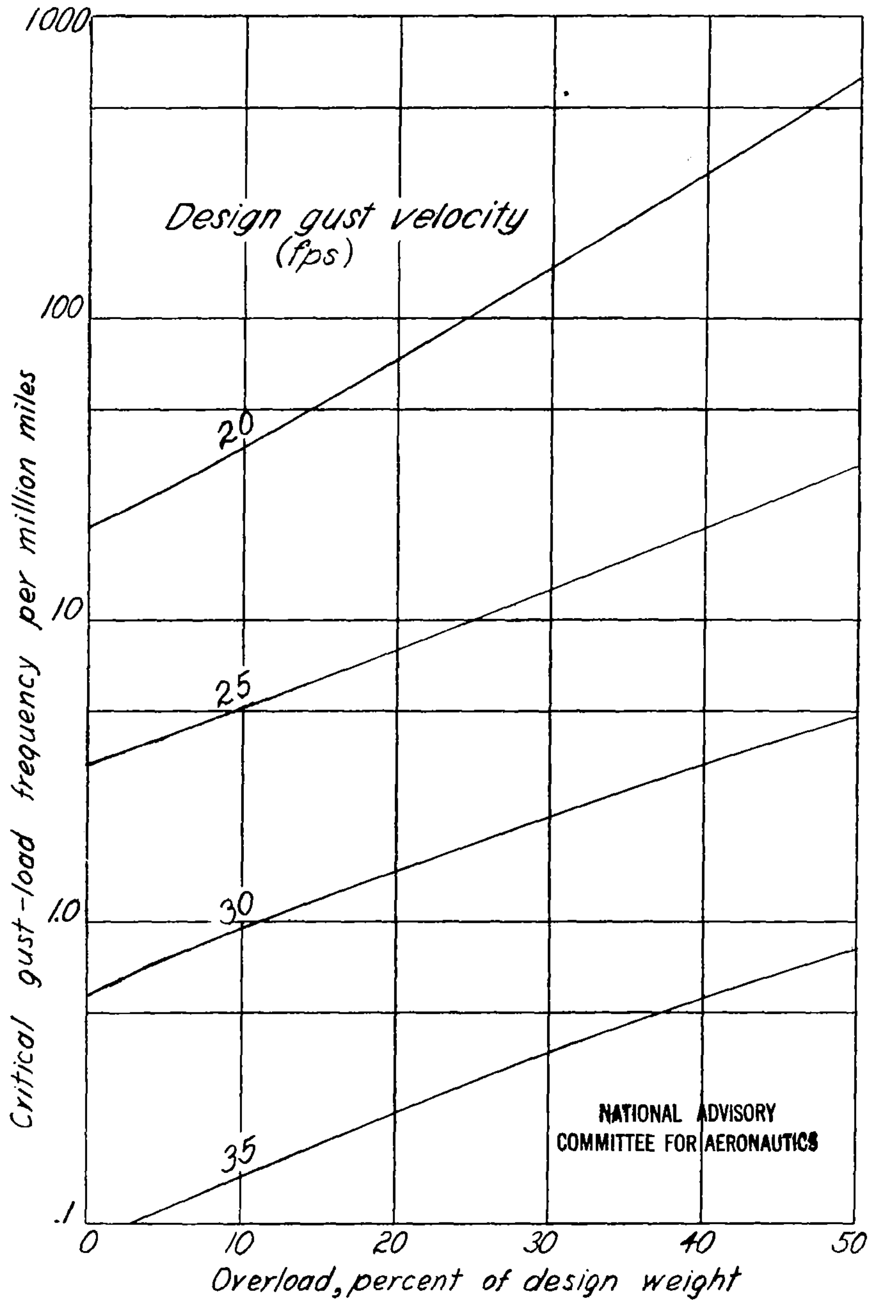


Figure 5.- Effect of design gust velocity on the critical gust-load frequency as a function of overload. Boeing S-307 airplane; case I.